

Independent
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Comparing
Means

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Koether

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Summary

Independent Samples: Comparing Means

Lecture 38
Section 11.4

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Outline

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Summary

- In this lecture, we will learn how to test a hypothesis concerning the difference between population means when σ is not known.
- We will also learn how to perform the test on the TI-83.

Proficiency in Statistics

The Concept of Proficiency in Statistics

Conceptual understanding

Comprehension of statistical concepts, operations, and relations

Procedural fluency

Skills in carrying out procedures flexibly, fluently, and appropriately

Strategic competence

Ability to formulate, represent, and solve statistical problems

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Summary

Adaptive reasoning

Capacity for logical thought, reflection, explanation, and justification

Productive disposition

Habitual inclination to see statistics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

The t Distribution

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Summary

- What if σ is unknown?
- Let s_1 and s_2 be the sample standard deviations.
- Whenever we use s_1 and s_2 instead of σ and the populations are normal, then we will have to use the t distribution instead of the standard normal distribution, unless the sample sizes are large.

Estimating σ

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Summary

- Individually, s_1 and s_2 estimate σ .
- However, we can get a better estimate for σ than either one of these if we “pool” them together.
- The pooled estimate is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$\bar{X}_1 - \bar{X}_2$ and the t Distribution

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Summary

- We should use t instead of Z if
 - The populations are normal,
 - We use s_p instead of σ , and
 - The sample sizes are small.
- The number of degrees of freedom is

$$df = df_1 + df_2 = n_1 + n_2 - 2.$$

- In this case

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

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Summary

- Work Example 11.4, p. 699 - Comparing Two Headache Treatments.
- State the hypotheses.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

- State the level of significance.

$$\alpha = 0.05.$$

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Summary

- The test statistic is

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with $df = n_1 + n_2 - 2$.

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Summary

- Compute

$$s_p = \sqrt{\frac{9s_1^2 + 9s_2^2}{18}} = 5.052.$$

and

$$t = \frac{22.6 - 19.4}{5.052 \sqrt{\frac{1}{10} + \frac{1}{10}}} = 1.416.$$

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Summary

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and

$$t = \frac{22.6 - 19.4}{5.052 \sqrt{\frac{1}{10} + \frac{1}{10}}} = 1.416.$$

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Summary

- Calculate the p -value.
- The number of degrees of freedom is $df = df_1 + df_2 = 18$.

$$\begin{aligned} p\text{-value} &= P(t > 1.416) \\ &= \text{tcdf}(1.416, E99, 18) \\ &= 0.0869. \end{aligned}$$

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Summary

- State the decision.
 - Accept H_0 .
- State the conclusion.
 - Treatment 1 is not more effective than Treatment 2.

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Summary

- The TI-83 will perform these tests.
 - Enter the data from the first sample into L_1 .
 - Enter the data from the second sample into L_2 .
 - Press `STAT > TESTS`.
 - Choose either `2-SampZTest` or `2-SampTTest`, depending on the circumstances.
 - Choose `Data` or `Stats`.
 - Provide the information that is called for.
 - `2-SampTTest` will ask whether to use a pooled estimate of σ . Answer “yes.”
 - Select `Calculate` and press `ENTER`.

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- The display shows, among other things, the value of the test statistic and the p -value.

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Summary

- Work Exercise 11.32 using the TI-83, assuming $\sigma = 6$.
- Work Example 11.4 using the TI-83.

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Summary

- Confidence intervals for $\mu_1 - \mu_2$ use the same theory.
- The point estimate is $\bar{x}_1 - \bar{x}_2$.
- The standard deviation of $\bar{x}_1 - \bar{x}_2$ is approximately

$$s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

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Summary

- The confidence interval is either

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

or

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

or

$$(\bar{x}_1 - \bar{x}_2) \pm t_{df, \alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

depending on the circumstances.

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Summary

- The choice depends on
 - Whether σ is known.
 - Whether the populations are normal.
 - Whether the sample sizes are large.

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Summary

- Find a 95% confidence interval for $\mu_1 - \mu_2$ in Example 11.4, p. 699.
- $\bar{x}_1 - \bar{x}_2 = 3.2$.
- $s_p = 5.052$.
- Use $t_{18,0.025} = 2.101$.
- The confidence interval is

$$3.2 \pm (2.101)(2.259) = 3.2 \pm 4.75.$$

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Summary

- Find a 95% confidence interval for $\mu_1 - \mu_2$ in Example 11.4, p. 699.
- $\bar{x}_1 - \bar{x}_2 = 3.2$.
- $s_p = 5.052$.
- Use $t_{18,0.025} = 2.101$.
- The confidence interval is

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Summary

- To find a confidence interval for the difference between means on the TI-83,
 - Press `STAT > TESTS`.
 - Choose either `2-SampZInt` or `2-SampTInt`, depending on circumstances.
 - Choose `Data` or `Stats`.
 - Provide the information that is called for.
 - `2-SampTTest` will ask whether to use a pooled estimate of σ . Answer “yes.”
 - Press `Calculate`.
- The confidence interval appears.

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Summary

- Find a 95% confidence interval for $\mu_1 - \mu_2$ in Example 11.4, p. 699 using the TI-83.

Summary

Summary

- If we use s_1 and s_2 instead of σ_1 and σ_2 , then the statistic has a t distribution with $n_1 + n_2 - 2$ degrees of freedom.
- The statistic is

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

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